Stability analysis with FEM: strain softening vs. strength reduction approach

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Abstract

In slope stability analysis, a progressive failure can occur due to a reduction of strength with increasing strain. This paper examines slope stability analysis with strain softening behaviour and their comparison with the strength reduction approach applying the finite element method. The results show that failure mechanisms obtained by both approaches are almost identical. The ratio between peak and reduced strength in strain softening behaviour is correlated with the factor of safety (FOS) in strength reduction technique.

For strain softening materials, it cannot be assumed that a safety factor (FOS) greater than one based on peak shear strength means stability, because deformations may lead to a local loss of strength, requiring mobilization of additional strength at other points along the slip surface. This mechanism leads to additional movement and further strain softening. Therefore, if the peak strength is mobilized anywhere along the failure surface, a slope in strain softening materials is at risk of progressive failure. Analysis of slope stability was done by the finite element method using the Mohr Coulomb failure criterion and an advanced Multilaminate model. In the proposed paper, the Multilaminate model for soil, enhanced with a non-local formulation as regularisation method, and a simple Mohr Coulomb model in invariant formulation are compared.

Introduction

The finite element method is an established numerical technique for solving boundary value problems in geotechnical engineering. Whereas in the past emphasis was put on calculating displacements and stresses under working load conditions the method is increasingly being applied to ultimate limit state calculations such as slope stability analysis because of its ability and flexibility. One of the advantages of finite element method as compared to traditional methods such as limit equilibrium methods is that no assumption of the shape and location of critical failure surfaces has to be made.

The main objective of this paper is to evaluate and to compare the failure mechanism of a simple slope obtained by a strain softening and strength reduction approach respectively and to check the influence of the stress path followed on the calculated factor of safety. Analysis of slope stability was performed by using a Mohr Coulomb and a Multilaminate model.

Shear Strength Reduction Technique

The shear strength reduction technique is considered the most widely used method in slope stability analysis using the finite element method. It works well when a simple Mohr

Coulomb model, which is a linear elastic perfect plastic model, where soil parameters are assumed to be constant during all stages of soil loading and unloading, is employed.

The principle of the shear strength reduction technique in finite element analysis is to simultaneously reduce c and tanφ in small increments until failure occurs in the numerical analysis. If shear strength parameters at failure are c_r and φ_r, the factor of safety (FOS) can be defined as:

$$FOS = \frac{\tan\varphi}{\tan\varphi_r} = \frac{c}{c_r} \tag{1}$$

Multilaminate Model with Strain Softening

In Multilaminate model, the constitutive behaviour is formulated on so called sampling planes (e.g. Schweiger *et al*., 2009). The analysis presented in this paper use the Multilaminate model with a linear strain softening formulation as developed by Galavi, 2007. In this formulation, the peak strength on a plane is reached at a certain strain level ($\varepsilon_{di,\ peak}$) followed by softening to a residual value $\varepsilon_{di,\ res}$ (**Figure 1** and **2**).

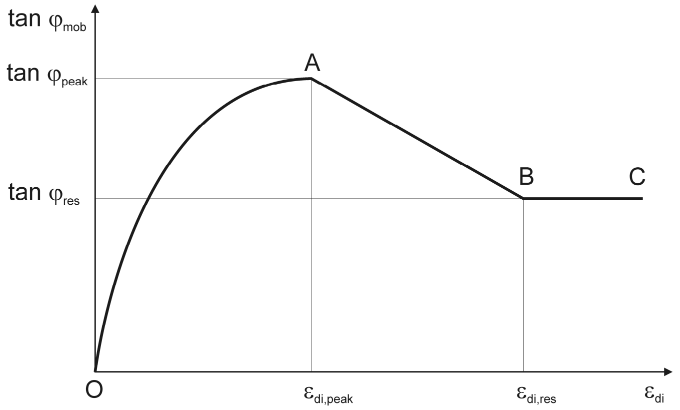


Figure 1: Relation between mobilized friction angle and damage strain on each sampling plane.

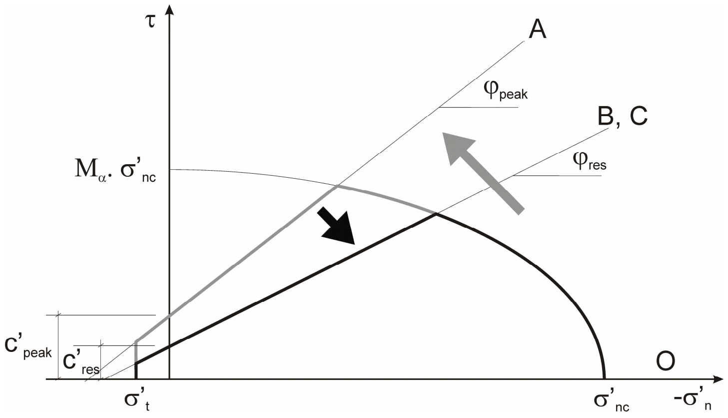


Figure 2: Deviatoric hardening and softening on a sampling plane.

Linear softening on integration planes is defined as:

$$\tan\varphi'_{mob} = -m_{soft,\varphi}\left(\varepsilon_{di} - \varepsilon_{di,peak}\right) + \tan\varphi'_{peak} \tag{2}$$

$$c'_{mob} = -m_{soft,c}\left(\varepsilon_{di} - \varepsilon_{di,peak}\right) + c'_{peak} \quad (3)$$

m_{soft} is the softening rate parameter that governs the reduction of the strength parameters on the sampling plane whereas in general cohesion degrades faster than the friction angle which reaches residual values after significant plastic deformation. Consequently, two independent softening rate parameters for cohesion ($m_{soft,c}$) and friction angle ($m_{soft,\varphi}$) have been introduced into the model.

Numerical Modelling of Slope Stability

In this section, slope stability analysis will be evaluated using four different methods to calculate the factor of safety (FOS):

1. Analysis with strain softening behaviour.
   In this method, slope stability analysis is carried out using the Multilaminate model with two different ratios of peak / residual strength (ratio of 1.1 and 1.3).
2. Analysis with reduced strength.
   In this method, slope stability analysis is carried out using the Multilaminate model with reduced strength. This reduced strength is obtained from two different ratios of peak / residual strength (ratio of 1.1 and 1.3)
3. Analysis with strength reduction in one step.
   In this method, slope stability analysis is carried out using the Multilaminate model with peak strength in the beginning of calculation. After the peak strength is reached, the material is changed to reduced strength. This leads to a stress state violating the strength criterion and thus to a redistribution of stresses. This reduced strength is also obtained from two different ratios of peak / residual strength (ratio of 1.1 and 1.3)
4. Analysis with “standard” strength reduction technique using the MC-criterion.
   In this method, slope stability analysis is carried out using the Mohr Coulomb model with the load obtained from the analysis with strain softening behaviour.

All analyses are performed utilizing the FE-code PLAXIS (Brinkgreve *et al.*, 2008),

Geometry, Finite Element Mesh and Material Properties

A simple geometry of a slope with homogeneous soil has been chosen. The slope is 10 m high and has a 1:2 gradient (horizontal to vertical). The geometry and finite element mesh used are shown in **Figure 3**. 458 15-noded elements have been used.

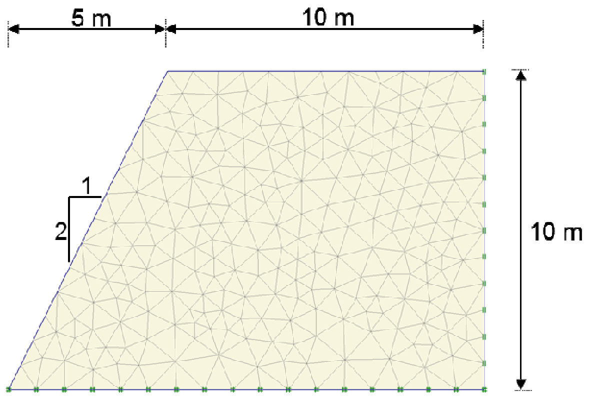


Figure 3: Geometry and finite element mesh of slope

The soil parameters for the Mohr Coulomb and the Multilaminate model are given in **Table 1** and **Table 2**.

Table 1: Input Parameters for Mohr Coulomb (MC) Model

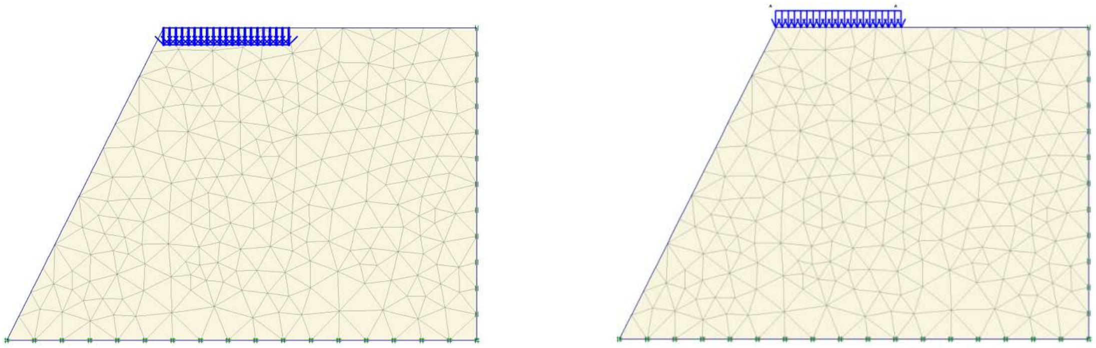
Description	***Symbol***	**Unit**	**Value**
Unit weight	γ	kN/m^3	20
Young modulus	E	(kPa)	10000
Poisson's ratio	ν	(-)	0.35
Cohesion	c	(kPa)	30
Friction angle	φ	(°)	35

Table 2: Input Parameters for Multilaminate (ML) Model

Description	**Symbol**	**Unit**	**Value**
unit weight	γ	kN/m^3	20
Reference oedometer modulus	$E_{oed_{ref}}$	(kPa)	10000
Reference young modulus for unloading and reloading	$E_{ur_{ref}}$	(kPa)	30000
Poisson's ratio	ν'	(-)	0.25
Peak cohesion	c'_{peak}	(kPa)	30
Peak friction angle	φ'_{peak}	(°)	35
Reduced cohesion (ratio of 1.1)	$c'_{res1.1}$	(kPa)	27.27
Reduced friction angle (ratio of 1.1)	$\varphi'_{res1.1}$	(°)	32.48
Reduced cohesion (ratio of 1.3)	$c'_{res1.3}$	(kPa)	23.08
Reduced friction angle (ratio of 1.3)	$\varphi'_{res1.3}$	(°)	28.31
Softening rate parameters for cohesion	$m_{soft,\,c}$	(-)	2.75
Softening rate parameters for friction angle	$m_{soft,\,\varphi}$	(-)	0.064

Boundary Conditions

In the analysis with the Multilaminate model, a prescribed displacement of 4 m width with a constant rate was imposed on the top of the slope. In Mohr Coulomb analysis, a load is imposed on the slope surface (**Figure 4**). The amount of loading is obtained from the residual force of the result of the Multilaminate model considering strain softening behaviour.



a) Prescribed displacement input in ML.

b) Load input in MC.

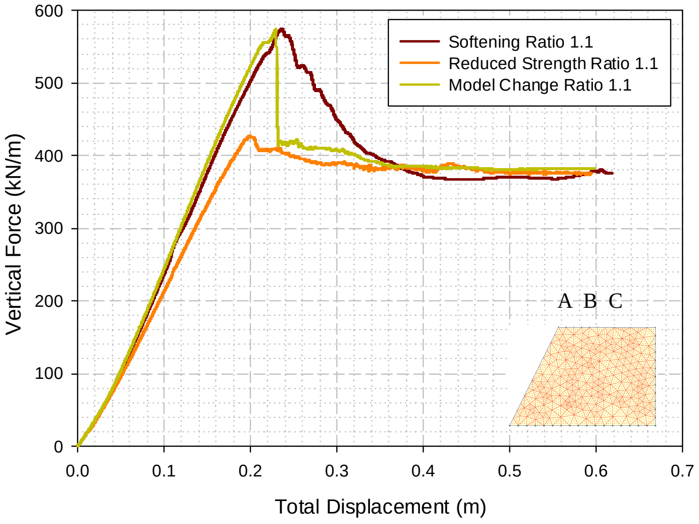
Figure 4: Loading conditions for the slope.

Results

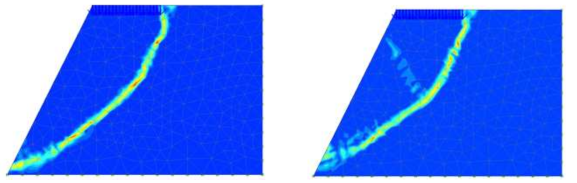
By applying the same ratio between peak and reduced strength (R) using the Multilaminate Model almost identical residual vertical forces are obtained whatever method is used (strain softening, reduced strength and change of strength in one step). The difference is only 2.5% with R = 1.1 and 11% with R = 1.3.

In the analysis slope stability with strain softening using the Multilaminate model with a ratio equal to 1.1, the residual vertical force at the surface of the slope is equal to 328.81 kN/m or 82.2 kN/m^2 (load along 4m at the surface). This value is applied as a load in the Mohr Coulomb model and gives a FOS of 1.08. When the ratio is increased to 1.3, the residual vertical force at the surface of the slope is equal to 231.30 kN/m or 57.8 kN/m^2 (load along 4m at the surface). If this value is applied as a load in the Mohr Coulomb model, a FOS of 1.21 is obtained. It should be mentioned that due to the formulation of the Multilaminate model, the strength is slightly higher as compared to the Mohr Coulomb model (with the some value for φ') and therefore some differences have to be expected.

All four different methods of calculation of the safety factor investigated (ML with strain softening, ML with reduced strength, ML with change of strength in one step and MC with strength reduction technique) seem to convergence to similar failure mechanisms and factors of safety. **Figure 5** and **6** show the vertical force – total displacement curves at the control point (A, B and C) of the top surface and failure mechanism for all analyses.

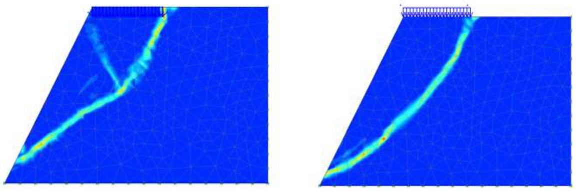


a) Vertical force vs. total displacement



b) ML model with softening analysis.

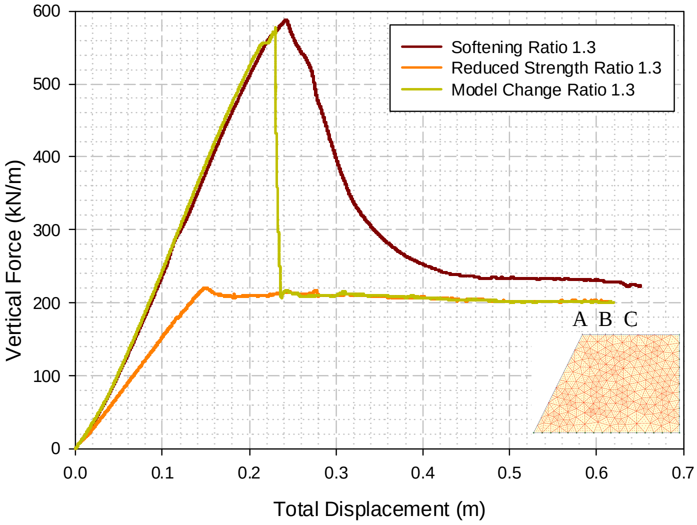
c) ML model with reduced strength.



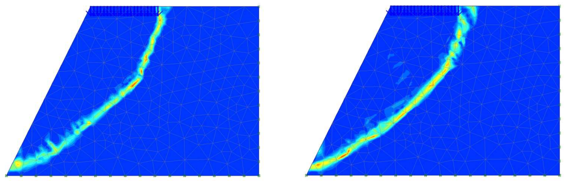
d) ML model with peak strength and reduction in 1 step

e) MC model with load based on softening analysis.

Figure 5: Analysis of slope stability with ratio peak / reduced strength equal to 1.1.

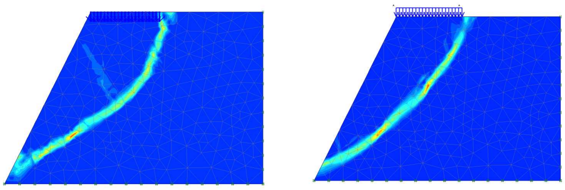


a) Vertical force vs. total displacement



b) ML model with softening analysis.

c) ML model with reduced strength.



d) ML model with peak strength and reduction in 1 step

e) MC model with load based on softening analysis.

Figure 6: Analysis of slope stability with ratio peak / reduced strength equal to 1.3.

Conclusion

Slope stability analysis based on the finite element method has been performed. Different methods of analysis with the Multilaminate model, namely (1) strain softening, (2) reduced strength from start of analysis, and (3) strength reduction in a single step to residual, yield very similar results with respect to failure mechanisms and factors of safety (FOS). Ratio of peak / residual strength in Multilaminate model with strain softening behaviour is close to FOS from strength reduction technique obtained from the Mohr Coulomb failure criterion.

Based on these (preliminary) studies it could be concluded (at least for such simple cases as considered here) that the stress paths followed are not crucial for calculating failure mechanisms and factors of safety. However, it is pointed out that only drained conditions and Mohr Coulomb type failure criteria have been considered so far.

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