Slope Stability Analysis with the Finite Element Method

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Abstract

In the evaluation of slopes, the factor of safety values still remain the main indexes for finding out how close or far slopes are from failure. The evaluation can be done my means of conventional slip circle analysis (the limit equilibrium methods) or by means of numerical methods such as the finite element method. This study presents the comparison between slope stability analysis with the finite element method and the limit equilibrium method.

Keyword: slope stability analysis, limit equilibrium method, finite element method.

Introduction

The development of soil and rock mechanics will influence the development of slope stability analyses in geotechnical engineering. Assessing the stability of engineered and natural slopes is a common challenge to both theoreticians and practitioners. The balance of natural slopes may be interrupted by man or nature causing stability problems. Natural slopes that have been stable for many years may suddenly fail due to changes in topography, seismicity, groundwater flows, loss of shear strength, stress change, and weathering (Abramson et al., 2002).

Duncan (1996) illustrated that the finite element method can be used to analyze the stability and deformations of slopes. Griffiths and Lane (1999) illustrated that the finite element method represents a powerful alternative method for slope stability analysis which is accurate, adaptable and requires less assumptions, especially concerning the failure mechanism. The failure mechanisms in the finite element method develop naturally through the regions wherein the shear strength of the soil is not sufficient to resist the shear stresses.

The main objective of this section is to evaluate and to compare the methods of slope stability analysis between limit equilibrium and finite element method by assuming a Mohr-Coulomb failure criterion.

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Limit Equilibrium Methods

Limit equilibrium methods are the most commonly used approaches in slope stability analysis. The fundamental assumption in these methods is that failure occurs through sliding of a mass along a slip surface. The reputation of the limit equilibrium methods is principally due to their relative simplicity, the ability to evaluate the sensitivity of stability to various input parameters, and the experience geotechnical engineer have acquired over the years in calculating the factor of safety.

The assumptions in the limit equilibrium methods are that the failing soil mass can be divided into slices and that forces act between the slices whereas different assumptions are made with respect to these forces in different methods. Some common features and limitation for equilibrium methods in slope stability analysis are summarized in Table 1. All methods use the same definition of the factor of safety:

$$FOS = \frac{Shear\ strength\ of\ soil}{Shear\ stress\ required\ for\ equilibrium} \tag{1}$$

The factor of safety is the factor by which the shear strength of the soil would have to be divided to carry the slope into a state of barely stable equilibrium.
The findings related to the accuracy of the limit equilibrium methods can be reviewed as follows:

1) For effective stress analysis of flat slopes, the ordinary method of slices is highly inaccurate. The computed factor of safety is too low. This method is accurate for ϕ = 0 analysis, and fairly accurate for any type of total stress analysis using circular slip surfaces.
2) For most conditions, the Bishop's modified method is reasonably accurate. Because of numerical problems, sometimes encountered, the computed factor of safety using the Bishop's modified method is different from the factor of safety for the same circle calculated using the ordinary method of slices.
3) Computed factor of safety using force equilibrium methods are sensitive to the assumption of the inclination of side forces between slices. A bad assumption concerning side force inclination will result in an inaccurate factor of safety.
4) Janbu's, Morgenstern and Prices's and Spencer's method that satisfy all conditions of equilibrium are accurate for any conditions. All of these methods have numerical problems under some conditions.

Table 1: Features and Limitation for Traditional Equilibrium Methods in Slope Stability Analysis (Duncan and Wright, 1980)

Method	Features and Limitation
Slope Stability Charts (Janbu, 1968, Duncan *et al*, 1987)	- Accurate enough for many purposes. - Faster than detailed computer analysis.
Ordinary Method of Slices (Fellenius, 1927)	- Only for circular slip surfaces. - Satisfies moment equilibrium. - Does not satisfy horizontal or vertical force equilibrium.
Bishop's Modified Method (Bishop, 1955)	- Only for circular slip surfaces. - Satisfies moment equilibrium. - Satisfies vertical force equilibrium. - Does not satisfy horizontal force equilibrium.
Force Equilibrium Methods (e.g. Lowe and Karafiath, 1960, Army Corps of Engineers, 1970)	- Any shape of slip surfaces. - Does not satisfy moment equilibrium. - Satisfies both vertical and horizontal force equilibrium.
Janbu's Generalized Procedure of Slices (Janbu, 1968)	- Any shape of slip surfaces. - Satisfies all conditions of equilibrium. - Permit side force locations to be varied. - More frequent numerical problems than some other methods.
Morgenstern and Price's Method (Morgenstern and Price, 1965)	- Any shape of slip surfaces. - Satisfies all conditions of equilibrium. - Permit side force orientations to be varied.
Spencer's Method (Spencer, 1967)	- Any shape of slip surfaces. - Satisfies all conditions of equilibrium. - Side forces are assumed to be parallel.

The limitation of limit equilibrium method in slope stability analysis has been demonstrated by Krahn (2003). This limitation is caused by the absence of a stress-strain relationship in the method of analysis. The limit equilibrium method lacks a suitable procedure for slope stability analysis under rapid loading condition as illustrated by Baker *et al.* (1993).

Finite Element Method

In the finite element method, the latter analysis, the so-called shear strength reduction (SSR) technique (Matsui & San 1992, Dawson *et al.* 1999) can be applied. The angle of dilatancy, soil modulus or the solution domain size are not critical parameters in this technique (Cheng, 1997). The safety factor can be obtained, assuming a Mohr-Coulomb failure criterion, by reducing the strength parameters incrementally, starting from unfactored values $\varphi_{available}$ and $c_{available}$, until no equilibrium can be found in the calculations. The corresponding strength parameters can be denoted as $\varphi_{failure}$ and $c_{failure}$ and the safety factor η_{fe} is defined as:

$$\eta_{fe} = \frac{\tan \varphi_{available}}{\tan \varphi_{failure}} = \frac{c_{available}}{c_{failure}} \tag{2}$$

There are two possibilities to arrive at the factor of safety as defined above.

Method 1: An analysis is performed with unfactored parameters modelling all construction stages required. The results represent the behaviour for working load conditions at the defined construction steps. This analysis is followed by an automatic reduction of strength parameters of the soil until equilibrium can be no longer achieved in the calculation. The procedure can be invoked at any construction step. This approach is sometimes referred to as φ/c-reduction technique.

Method 2: The analysis is performed with factored parameters from the outset, i.e. strength values are reduced, again in increments, but a new analysis for all construction stages is performed for each set of parameters. If sufficiently small increments are used the factor of safety is again obtained from the calculation where equilibrium could not be achieved.

Both methods are straightforward to apply when using a standard Mohr-Coulomb failure criterion. In the finite element method, failure occurs naturally through the zones within the soil mass wherein the shear strength of the soil is not capable to resist the applied shear stress, so there is no need to make assumption about the shape or location of the failure surface.

Mohr-Coulomb Failure Criterion

The Mohr-Coulomb failure criterion is commonly used to describe the strength of soil. The relationship between shear strength and the principal stresses active on a mass of soil can be represented in terms of the Mohr circle of stress where the limits on the principal stress axis represent the major and minor principal stresses, σ_1 and σ_3. Mohr-Coulomb's failure criterion (Figure 1) is a line forming a tangent to the circle at point *a*. The slope of this line is the friction angle, φ, and the line intercepts the shear stress axis at the value of the soil's cohesion, *c*.

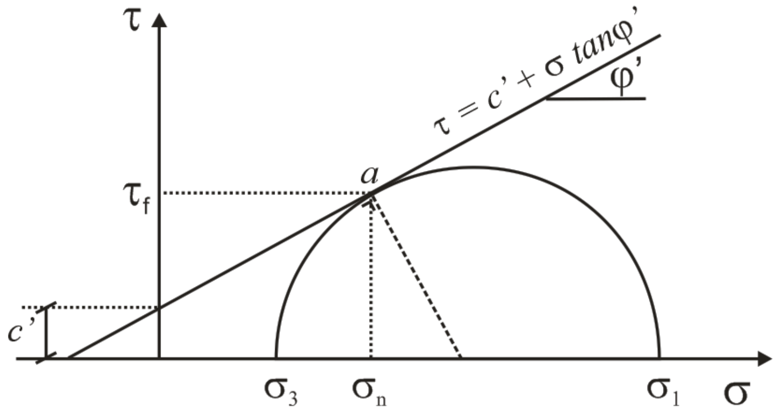

Figure 1: Mohr-Coulomb failure criterion

So, the Mohr-Coulomb failure envelope may be described by:

$$\tau = c' + \sigma' \tan\varphi' \tag{3}$$

Alternatively the Mohr-Coulomb criterion can be formulated in terms of principal stresses as follows:

$$f_{1a} = \tfrac{1}{2}(\sigma'_2 - \sigma'_3) + \tfrac{1}{2}(\sigma'_2 + \sigma'_3)\sin\varphi - c\cos\varphi \le 0 \tag{4}$$

$$f_{1b} = \tfrac{1}{2}(\sigma'_3 - \sigma'_2) + \tfrac{1}{2}(\sigma'_3 + \sigma'_2)\sin\varphi - c\cos\varphi \le 0 \tag{5}$$

$$f_{2a} = \tfrac{1}{2}(\sigma'_3 - \sigma'_1) + \tfrac{1}{2}(\sigma'_3 + \sigma'_1)\sin\varphi - c\cos\varphi \le 0 \tag{6}$$

$$f_{2b} = \tfrac{1}{2}(\sigma'_1 - \sigma'_3) + \tfrac{1}{2}(\sigma'_1 + \sigma'_3)\sin\varphi - c\cos\varphi \le 0 \tag{7}$$

$$f_{3a} = \tfrac{1}{2}(\sigma'_1 - \sigma'_2) + \tfrac{1}{2}(\sigma'_1 + \sigma'_2)\sin\varphi - c\cos\varphi \le 0 \tag{8}$$

$$f_{3b} = \tfrac{1}{2}(\sigma'_2 - \sigma'_1) + \tfrac{1}{2}(\sigma'_2 + \sigma'_1)\sin\varphi - c\cos\varphi \le 0 \tag{9}$$

Figure 2 illustrates a fixed hexagonal cone in principal stress space with the condition $f_i = 0$ for all yield function. The Mohr-Coulomb plastic potential functions that contain a third plasticity parameter, the so-called dilatancy angle ψ are given by:

$$g_{1a} = \tfrac{1}{2}(\sigma'_2 - \sigma'_3) + \tfrac{1}{2}(\sigma'_2 + \sigma'_3)\sin\psi \tag{10}$$

$$g_{1b} = \tfrac{1}{2}(\sigma'_3 - \sigma'_2) + \tfrac{1}{2}(\sigma'_3 + \sigma'_2)\sin\psi \tag{11}$$

$$g_{2a} = \tfrac{1}{2}(\sigma'_3 - \sigma'_1) + \tfrac{1}{2}(\sigma'_3 + \sigma'_1)\sin\psi \tag{12}$$

$$g_{2b} = \tfrac{1}{2}(\sigma'_1 - \sigma'_3) + \tfrac{1}{2}(\sigma'_1 + \sigma'_3)\sin\psi \tag{13}$$

$$g_{3a} = \tfrac{1}{2}(\sigma'_1 - \sigma'_2) + \tfrac{1}{2}(\sigma'_1 + \sigma'_2)\sin\psi \tag{14}$$

$$g_{3b} = \tfrac{1}{2}(\sigma'_2 - \sigma'_1) + \tfrac{1}{2}(\sigma'_2 + \sigma'_1)\sin\psi \tag{15}$$

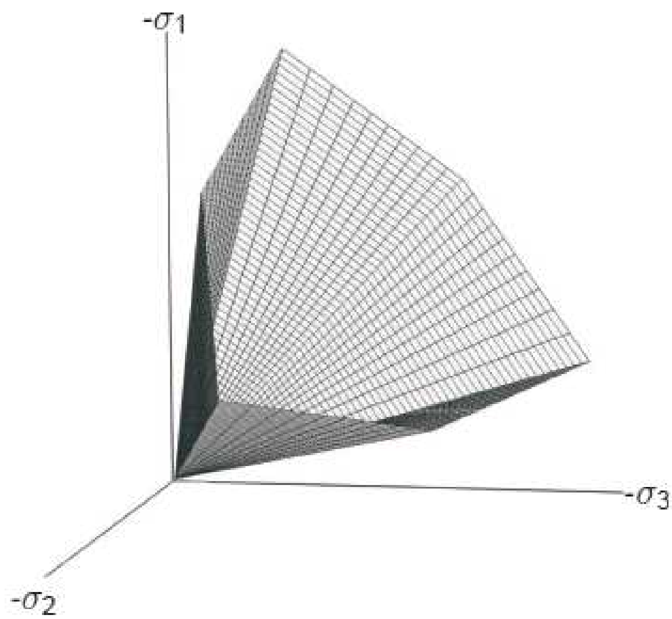

Figure 2: The Mohr-Coulomb yield surface (c = 0) (Brinkgreve *et al*, 2010)

Generally, the linear perfectly-plastic Mohr-Coulomb model requires five parameter. These parameters are the Young's modulus (E), Poisson's ratio (ν), cohesion (c), friction angle (φ), and dilatancy angle (ψ).

Slope Stability Examples

In this section, five examples of slope stability analysis from Griffiths and Lane (1999) will be discussed. These examples will be analyzed by the finite element method and will be compared with the limit equilibrium methods. The analysis was performed by utilizing PLAXIS for the finite element method and Slope/W for the limit equilibrium methods. The soil model used in the analysis is the Mohr-Coulomb failure criterion.

Homogeneous slope with no foundation layer

The height of the homogeneous slope is 10 m and the gradient (horizontal to vertical) is 2:1. **Figure 3** illustrates the geometry and the two dimensional finite element meshes consisting of 390 15-noded elements.

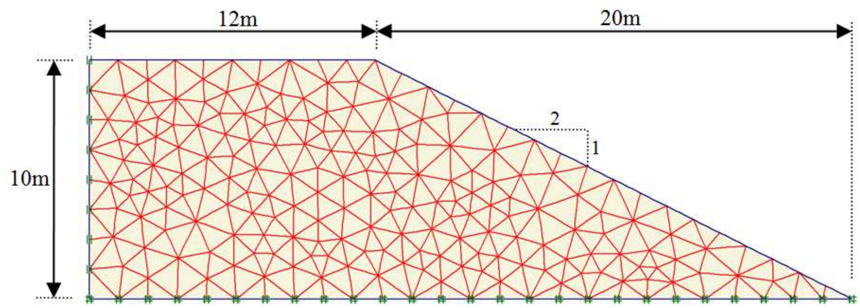

Figure 3: Geometry and mesh for a homogeneous slope with no foundation layer

The soil parameters for this example are given in Table 2.

Table 2: Soil Parameters for Example 1 with Mohr Coulomb Model

Description	Symbol	Unit	Value
Unit weight	γ	[kN/m^3]	20
Effective young's modulus	E’	[kPa]	100000
Effective poisson's ratio	ν'	[-]	0.3
Cohesion (effective shear strength)	c'	[kPa]	10
Friction angle (effective shear strength)	ϕ'	[^{o}]	20

In Figure 4 the failure mechanism for this example is presented for the finite element method. The result of the slope stability analysis using a limit equilibrium method is presented in Figure 5.

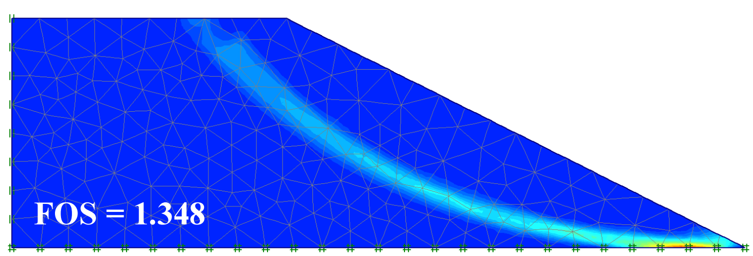

Figure 4: Failure mechanism for a homogeneous slope with no foundation using the finite element method

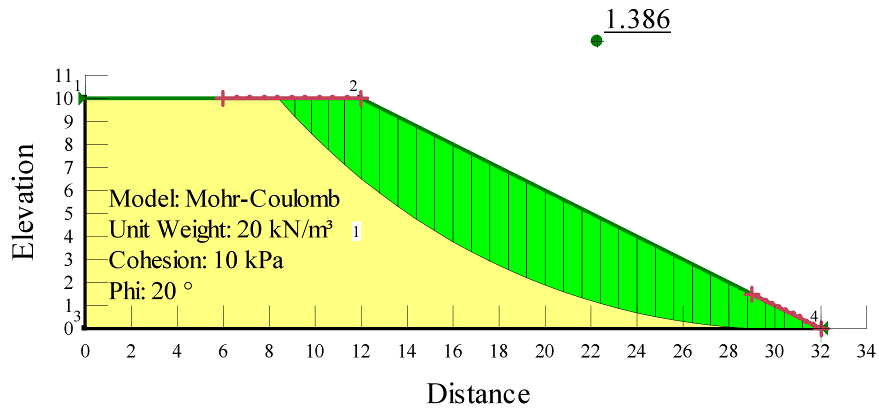

Figure 5: Morgenstern and Price Method for a homogeneous slope with no foundation

The difference of factor of safety between the finite element method and limit equilibrium methods is only 2.8% and the failure mechanism are similar.

Homogeneous slope with a foundation layer

The homogeneous slope has a foundation layer with the thickness of half of the slope height in this example. The height of the slope is 10 m and the gradient (horizontal to vertical) is 2:1. Figure 6 shows the geometry and the two dimensional finite element mesh consisting of 309 15-noded elements.

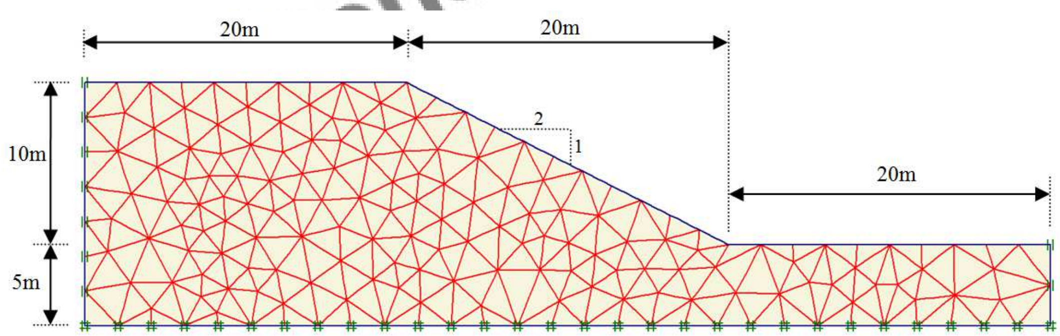

Figure 6: Geometry and mesh for a homogeneous slope with a foundation layer

The soil parameters used for this example are the same as given in Table 2. In Figure 7 the failure mechanism is presented for the finite element method and the result of the analysis using a limit equilibrium method is presented in Figure 8.

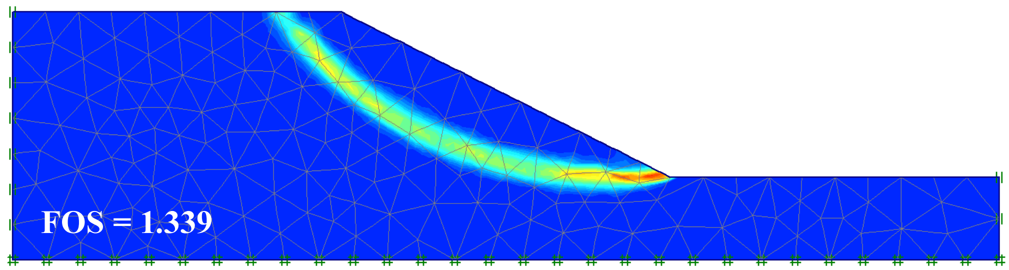

Figure 7: Failure mechanism for a homogeneous slope with a foundation layer using the finite element method

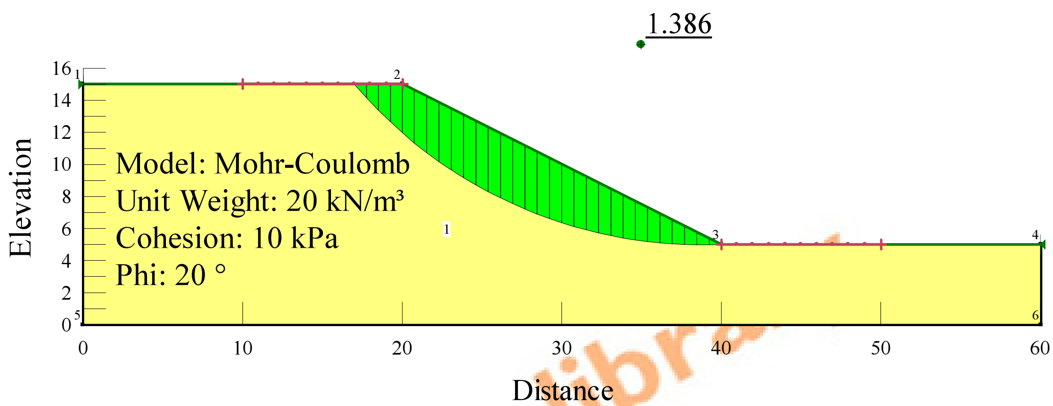

Figure 8: Morgenstern and Price Method for a homogeneous slope with a foundation layer

The difference of factor of safety between the finite element method and limit equilibrium methods is only 3.5% and the computed failure mechanism are similar.

An undrained clay slope with a thin weak layer

Figure 9 shows the geometry and the two dimensional finite element mesh of the example of an undrained clay slope with a thin weak layer. The height of the slope is 10 m and the slope is inclined at an angle of 26.57^{o} (2:1) to the horizontal. The mesh consist of 562 15-noded elements.

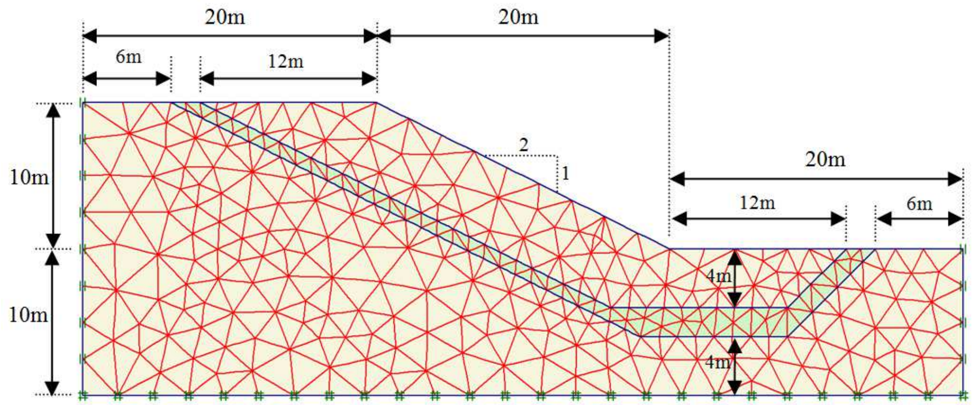

Figure 9: Geometry and mesh for an undrained clay slope with a thin weak layer

The soil parameters for this example are given in Table 3. The analyses are carried out using a constant value of undrained shear strength of the soil (c_{u1}) and five different values of undrained shear strength of the thin layer (c_{u2}) with ratios c_{u2}/c_{u1} equal to 1, 0.8, 0.6, 0.4, and 0.2.

Table 3: Soil Parameters for Example 3 with Mohr Coulomb Model

Description	Symbol	Unit	Value
Unit weight	γ	[kN/m^3]	20
Effective young's modulus	E'	[kPa]	100000
Effective poisson's ratio	ν'	[-]	0.3
Cohesion (undrained shear strength)	c_{u1}	[kPa]	50
Friction angle (undrained shear strength)	ϕ_u	[^{o}]	0

In Figure 10 computed failure mechanisms for this example are presented for the finite element method with different ratios c_{u2}/c_{u1} and the result of the analysis using a limit equilibrium method is presented in Figure 11.

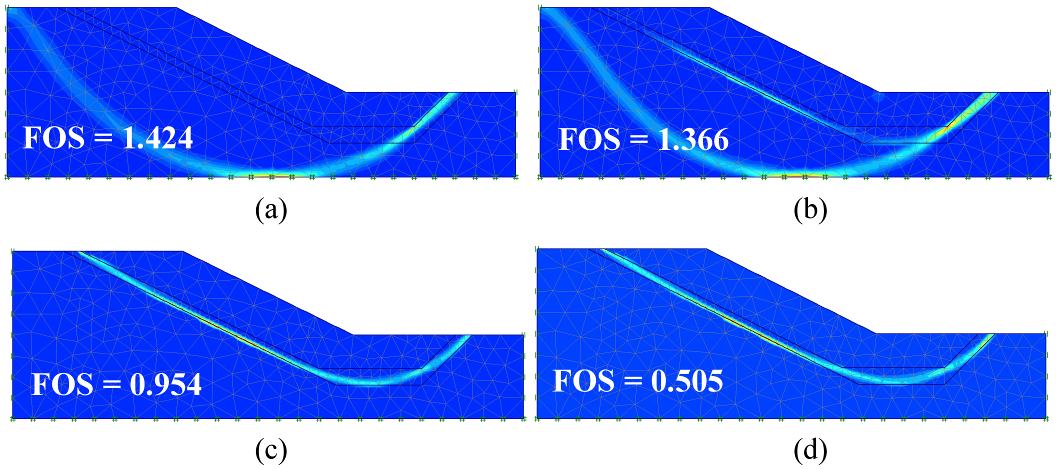

Figure 10: Failure mechanism for an undrained clay slope with a thin weak layer using Finite Element Method; (a) $c_{u2}/c_{u1} = 0.8$; (b) $c_{u2}/c_{u1} = 0.6$; (c) $c_{u2}/c_{u1} = 0.4$; (d) $c_{u2}/c_{u1} = 0.2$

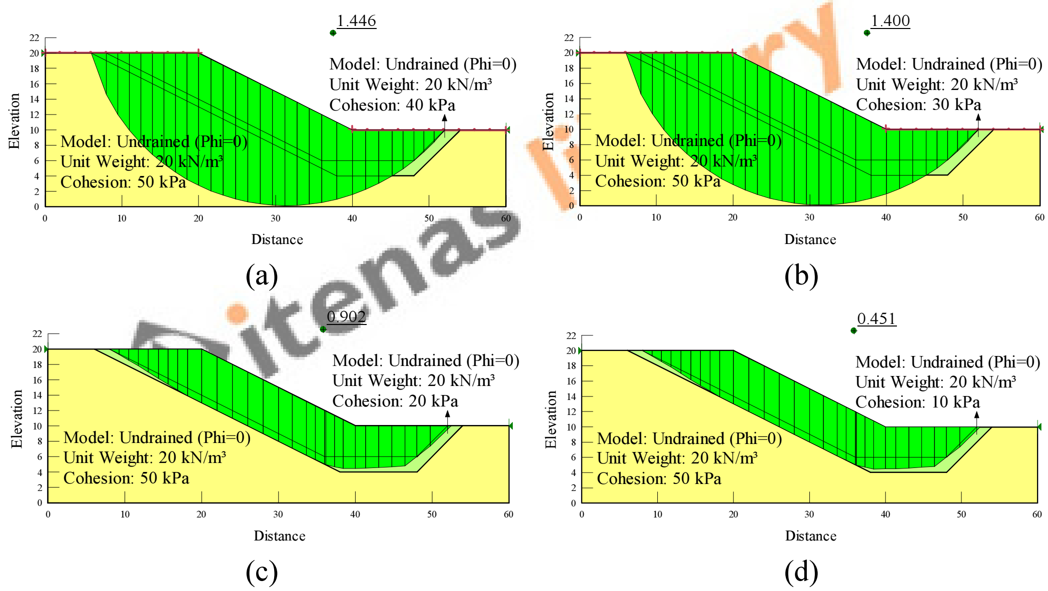

Figure 11: Morgenstern and Price Method for an undrained clay slope with a thin weak layer; (a) $c_{u2}/c_{u1} = 0.8$; (b) $c_{u2}/c_{u1} = 0.6$; (c) $c_{u2}/c_{u1} = 0.4$; (d) $c_{u2}/c_{u1} = 0.2$

The factor of safety obtained using the finite element and limit equilibrium methods for this example are summarised in Table 4 and illustrated in Figure 12.

Table 4: Computed factor of safety for Example 3

c_{u2}/c_{u1}	FOS	
	Finite Element Method	Morgenstern and Price Method
1.0	1.451	1.448
0.8	1.424	1.446
0.6	1.366	1.400
0.4	0.954	0.902
0.2	0.505	0.451

The computed factor of safety with the ratio $c_{u2}/c_{u1} > 0.6$ using the finite element method are close to the Morgenstern and Price method and the failure mechanisms of these methods are similar. With these ratios, the strength of the thin weak layer does not affect the safety factor of the slope and generate a circular (base) mechanism of failure. When the ratio c_{u2}/c_{u1} reduced to 0.6, the finite element method produce two failure mechanisms. The first failure mechanism is a base mechanism combined with the weak layer beyond the slope toe and the second failure mechanism is a non-circular mechanism closely following the geometry of the thin weak layer. This is shown in Figure 10. With this ratio, the Morgenstern and Price method only produce one failure mechanism, the so-called circular (base) mechanism. When the ratio c_{u2}/c_{u1} is reduced to 0.4 and 0.2, the failure mechanism of the slope shows a non-circular mechanism closely following the geometry of the thin weak layer. However, with the ratio $c_{u2}/c_{u1} \leq 0.6$, there is no significant difference of factor of safety between the finite element method and the Morgenstern and Price method.

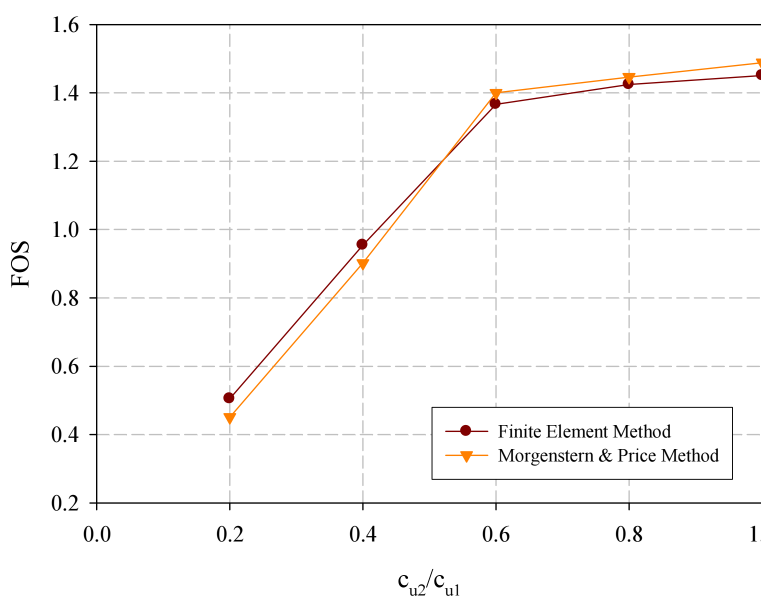

Fig. 12: Computed FOS for an undrained clay slope with a thin weak layer with variations of c_{u2}/c_{u1}

An undrained clay slope with a weak foundation layer

In this example analysis of an undrained clay slope of 10m height and a 10m thick foundation layer is carried out. The slope is inclined at an angle of 26.57° (2:1) to the horizontal. Figure 13 shows the geometry and the two dimensional finite element mesh consisting of 562 15-noded elements.

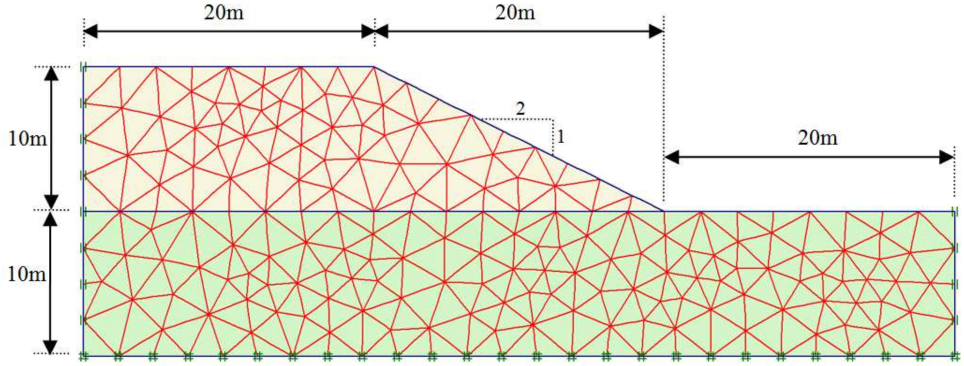

Figure 13: Geometry and mesh for an undrained clay slope with a weak foundation layer

The soil parameters used for this example are given in Table 3. The analysis are carried out using a constant value of undrained shear strength of soil (c_{u1}) and six different values of undrained shear strength of the foundation layer (c_{u2}) with ratios c_{u2}/c_{u1} equal to 0.5, 1.0, 1.5, 1.75, 2.0 and 2.5.

In Figure 14 computed failure mechanisms for this example are presented for the finite element method and the results of slope stability analysis using limit equilibrium methods are presented in Figure 15.

The factor of safety obtained using the finite element and limit equilibrium methods for this example are summarised in Table 5 and illustrated in Figure 16. The average difference of factor of safety between the finite element method and limit equilibrium methods is only 2.2% and the failure mechanisms of these methods are similar except when the ratio c_{u2}/c_{u1} = 1.5. When the ratio c_{u2}/c_{u1} =1.5, the finite element method generates two failure mechanisms, namely a base mechanism and a toe mechanism. It represents the transition between these two fundamental mechanisms. However, with this ratio, the Morgenstern and Price Method only generates one failure mechanism, the so-called toe mechanism.

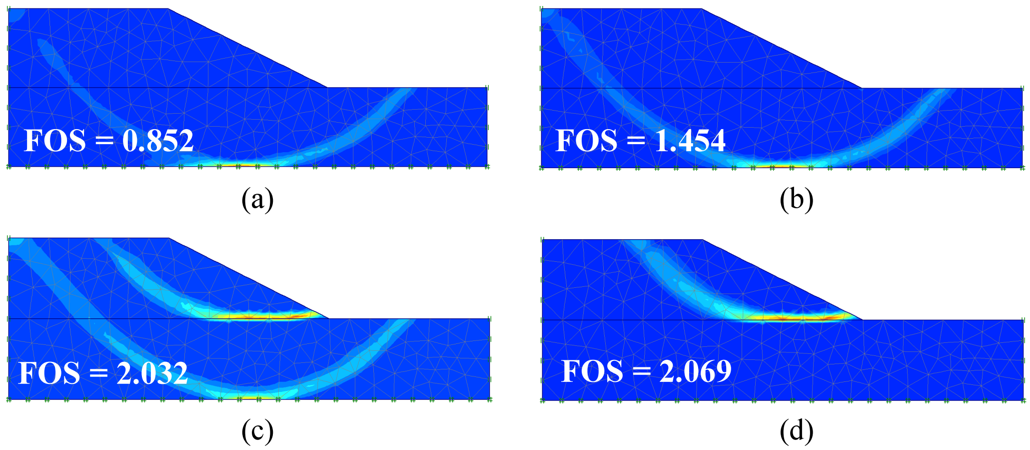

Figure 14: Failure mechanism for an undrained clay slope with a weak foundation layer using the finite element method; (a) c_{u2}/c_{u1} = 0.5; (b) c_{u2}/c_{u1} = 1.0; (c) c_{u2}/c_{u1} = 1.5; (d) c_{u2}/c_{u1} = 1.75

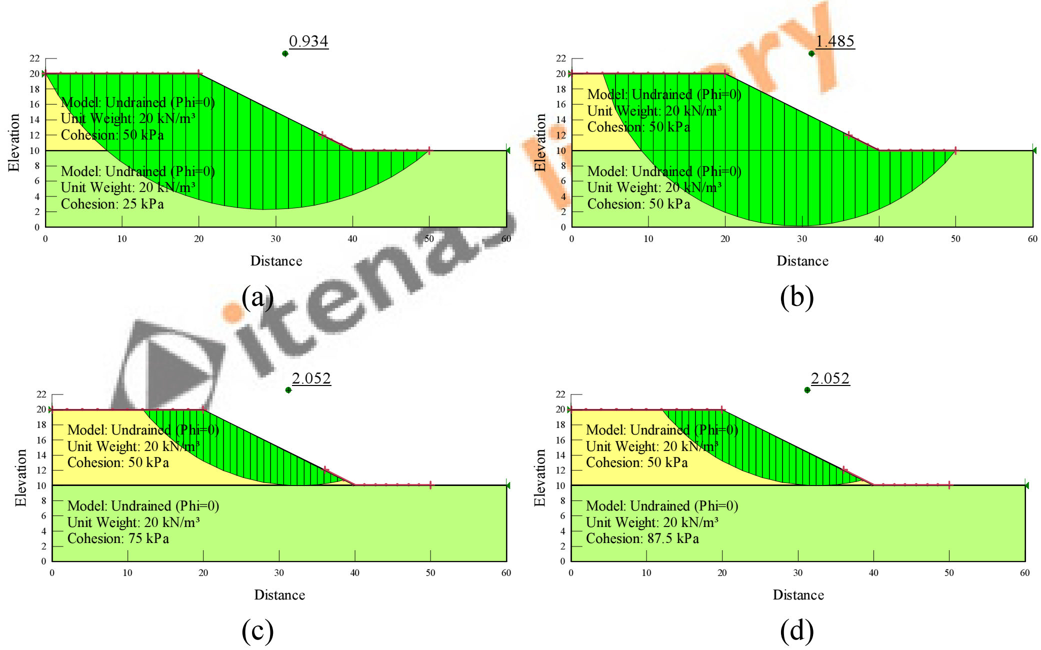

Figure 15: Morgenstern and Price Method for an undrained clay slope with a weak foundation layer; (a) c_{u2}/c_{u1} = 0.5; (b) c_{u2}/c_{u1} = 1.0; (c) c_{u2}/c_{u1} = 1.5; (d) c_{u2}/c_{u1} = 1.75

Table 5: Computed factor of safety for Example 4

c_{u2}/c_{u1}	FOS	
	Finite Element Method	Morgenstern and Price Method
0.50	0.852	0.934
1.00	1.454	1.485
1.50	2.032	2.052
1.75	2.069	2.052
2.00	2.076	2.064
2.50	2.069	2.064

Figure 16 shows that at c_{u2}/c_{u1} = 1.5 the factor of safety remains constant for both methods.

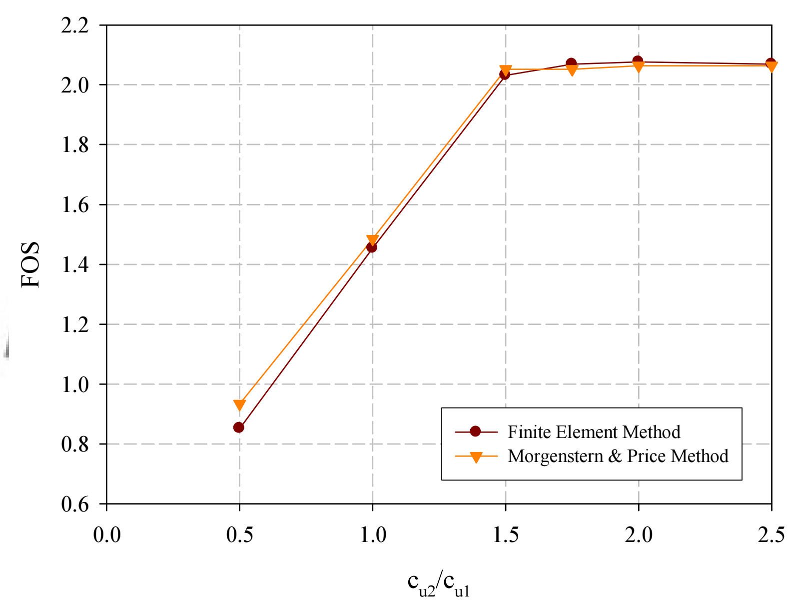

Fig. 16: Computed FOS for an undrained clay slope with a weak foundation layer with variations of c_{u2}/c_{u1}

Homogeneous slope with water level

The geometry, the two dimensional finite element mesh and the soil parameters of this example are the same as the slope analysed in Example 1, combined with a water level at a depth *L* below the crest of the slope (Figure 17).

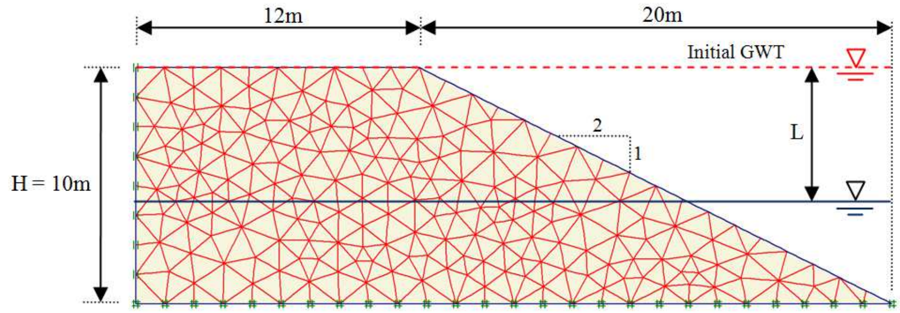

Figure 17: Geometry and mesh for homogeneous slope with water level

In this analysis, a slope with different drawdown ratios L/H, which has been varied from 0.0 (slope completely submerged with water level at the crest of the slope) to 1.0 (water level at the toe of the slope) is considered. This example is the so-called slow drawdown problem wherein a reservoir, initially at the crest of the slope, is slowly lowered to the base, with the water level within the slope maintaining the same level.

Figure 18 shows the computed failure mechanisms for this example using the finite element method and Figure 19 illustrates the results of slope stability analysis using limit equilibrium methods.

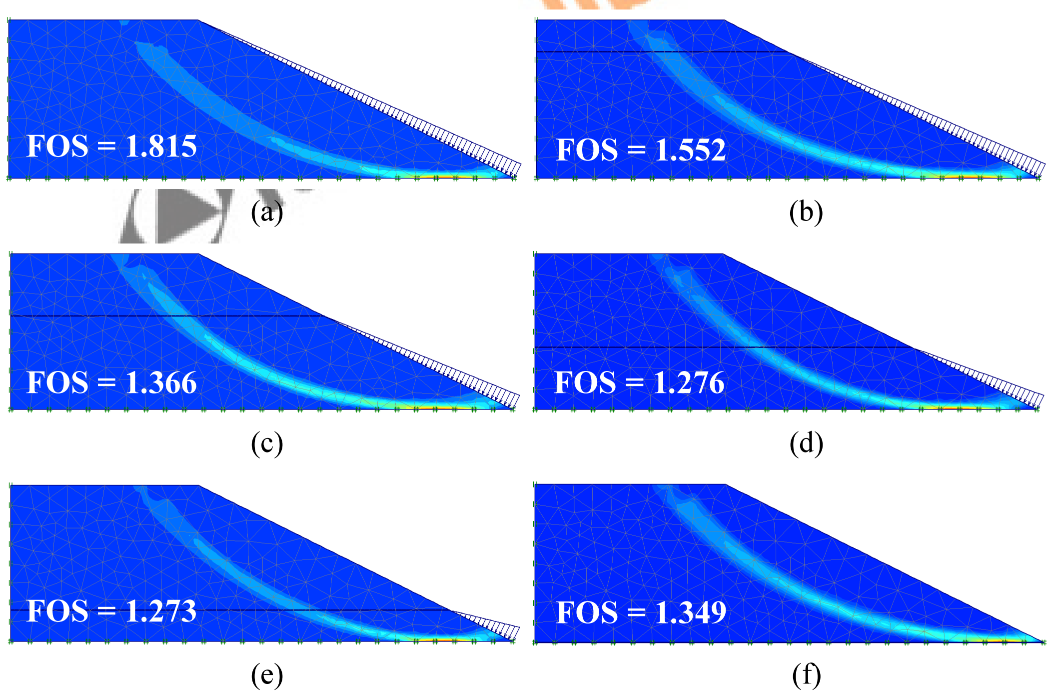

Figure 18: Failure mechanism for homogeneous slope with water level using the finite element method; (a) L/H = 0.0; (b) L/H = 0.2; (c) L/H = 0.4; (d) L/H = 0.6; (e) L/H = 0.8; (f) L/H = 1.0

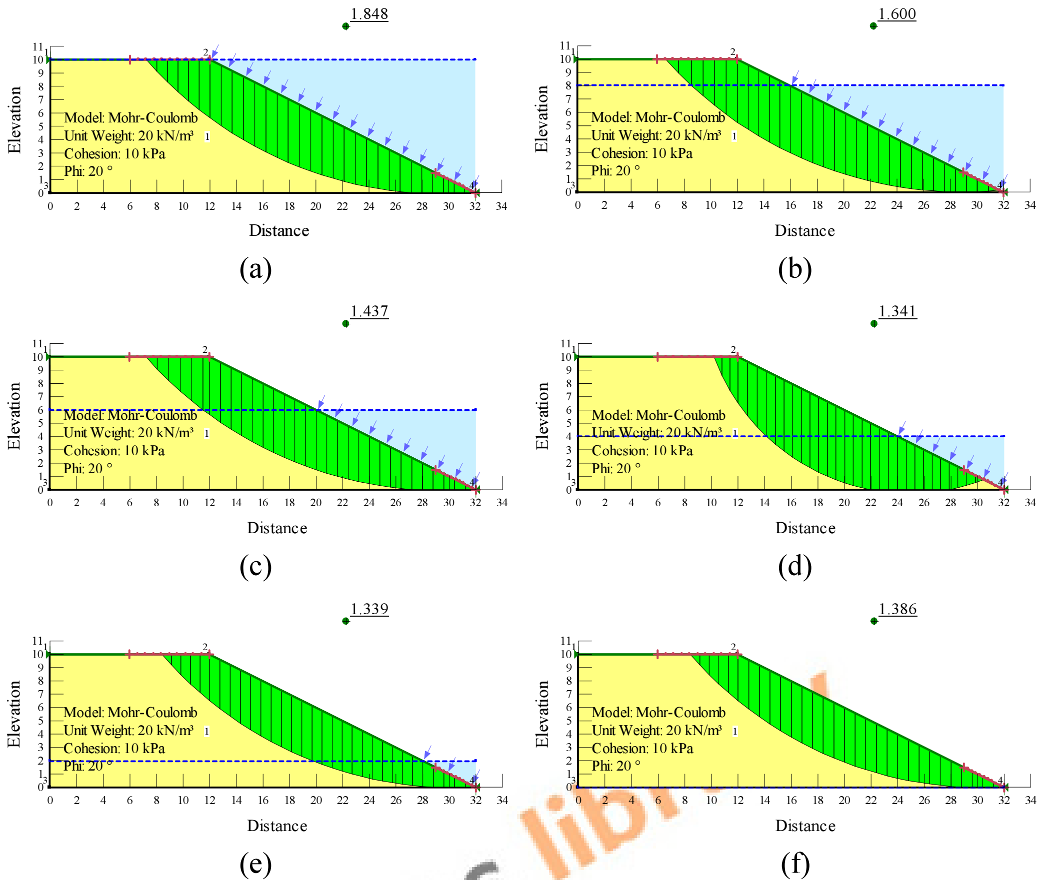

Figure 19: Morgenstern and Price Method for homogeneous slope with horizontal water level (L/H = 1.0); (a) L/H = 0.0; (b) L/H = 0.2; (c) L/H = 0.4; (d) L/H = 0.6; (e) L/H = 0.8; (f) L/H = 1.0

The factor of safety obtained using the finite element and limit equilibrium methods for this example are summarised in Table 6 and illustrated in Figure 20. The average difference of factor of safety between the finite element method and limit equilibrium methods is only 3.7% and the failure mechanisms of these methods are similar.

In fully slow drawdown conditions, the factor of safety reaches a minimum when L/H = 0.7 and the fully submerged slope (L/H = 0) is more stable than the dry slope (L/H = 1) as indicated by a higher factor of safety, which has been also demonstrated by Lane and Griffith (2000). The most severe condition is not when the water level was lowered to a minimum. It was observed that the movement near the toe was significantly upward and the failure mechanism changed when the water level was lowered to the base.

Table 6: Computed factor of safety for Example 5

L/H	FOS	
	Finite Element Method	Morgenstern and Price Method
-0.1	1.815	1.847
0.0	1.815	1.858
0.1	1.685	1.715
0.2	1.552	1.600
0.3	1.449	1.507
0.4	1.366	1.437
0.5	1.308	1.378
0.6	1.276	1.341
0.7	1.259	1.331
0.8	1.273	1.339
0.9	1.305	1.356
1.0	1.349	1.386

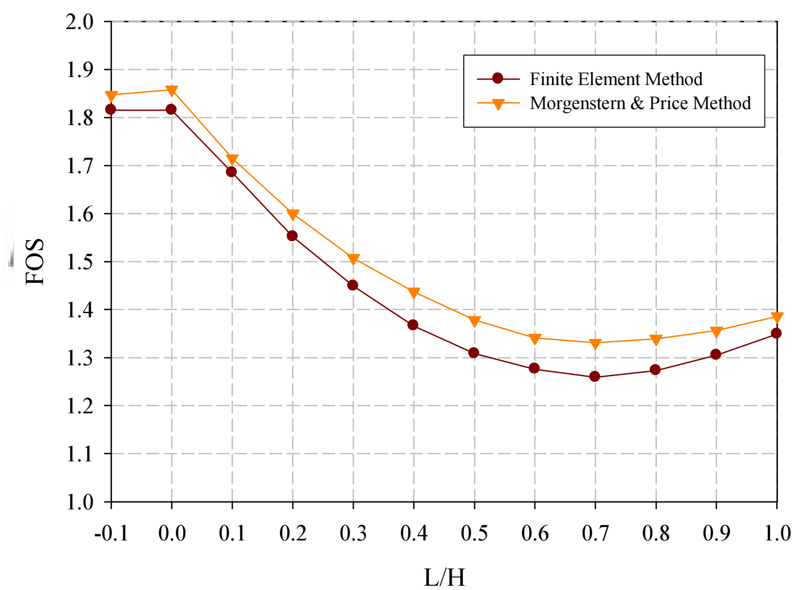

Fig. 20: Computed FOS for homogeneous slope with variations of L/H

Summary

The two approaches of slope stability analyses, one based on limit equilibrium methods and the other based on the finite element method are widely used in geotechnical engineering. The finite element method in combination with an elastic-perfectly plastic (Mohr-Coulomb) model has been shown to be suitable for slope stability analysis.

In simple cases similar factors of safety and failure mechanism are obtained as in limit equilibrium analysis, however under more complex conditions the finite element method is more versatile because no a priori assumptions on the shape of the failure mechanism has to be made.

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